

He has constructed a large number of variants which deal with different situations. These include those in which there is a weighting function  $e^{px}$ , those in which end-points do not coincide with grid points and those in which endpoint connections at these inconvenient points are expressed in terms of derivatives. The weighting function  $(x - x_0)^a$  is also treated but on a less ambitious scale. The coefficients in this case involve Riemann zeta functions and related functions. These are tabulated and in general the user has to resort to numerical interpolation to calculate the coefficients. The finite-difference enthusiast will surely find many new and complicated expansions here.

As is conventional in this field there is no discussion about the convergence of these expansions. Numerical examples are chosen from the rather limited subset of functions for which the expansions happen to converge.

This reviewer found one part of the book to be of more general interest. The author derives a generalization of the Euler-Maclaurin summation formula. When  $F(x) = e^{px}f(x)$ , the asymptotic expansion on the right-hand side in the conventional form

$$h \sum_{r=0}^{n-1} F(x_r + sh) - \int_{x_0}^{x_n} F(t) dt \simeq \sum_{r=0}^N h^{r+1} \frac{B_{r+1}(s)}{(r+1)!} \{F^{(r)}(x_n) - F^{(r)}(x_0)\} + O(h^{N+2})$$

may be transformed into

$$\sum_{r=0}^N h^{r+1} \frac{e^{sph} D_r(s, ph)}{r!} \{e^{px_n} f^{(r)}(x_n) - e^{px_0} f^{(r)}(x_0)\} + O(h^{N+2}),$$

where the function  $D_r(s, q)$  has many interesting elementary properties, some of which are quite tricky. Some of these are generalizations of analogous properties of the Bernoulli functions and the Euler functions. The author presents an excellent account of this function. In an appendix the author establishes the close connection between this function and the higher transcendental function

$$\phi(z, -r, s) = \sum_{j=0}^{\infty} (s + j)^r z^j.$$

The section about the  $D$ -functions would have certainly reached a wider and responsive audience if it had been published as a paper in a journal. However, the bulk of the book serves mainly as a repository for finite-difference expansions.

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**39[2.20].**—J. H. GARCIA RODRIGUEZ & F. REVERON OSIO, *Tablas para la Resolucion de Ecuaciones de Tercero y Quinto Grado* (Tables for the Solution of Equations of the Third and Fifth Degree), Universidad de los Andes, Facultad de Ingenieria, Escuela de Ingenieria Electrica, Merida, Venezuela, 1968, 201 pp.

For solving the general cubic  $Ay^3 + By^2 + Cy + D = 0$ , after it is reduced to

$$(1) \quad X^3 + pX + q = 0$$

by the transformation  $y = X - B/3A$ , the authors give, on pp. 17–176, tables for the roots of (1) for  $p = -100(1)100$ ,  $q = 0(1)100$ , to 5D (6S for about 70 percent of the

entries). For negative  $q$ , let  $X = -X'$  in (1). When (1) has a pair of complex roots  $R \pm I$ , the authors tabulate  $R$  and  $I$  on opposite sides of a page, leaving the third root  $-2R$  for the reader to find. When (1) has three real roots, the authors tabulate two,  $X_1$  and  $X_2$ , on opposite sides of a page, leaving the third,  $X_3 = -(X_1 + X_2)$ , for the reader to find.

For solving the general quintic  $ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$ , expressible as  $ay^5 + gy^3 + hy^2 + ky + m = 0$  after the transformation  $x = y - b/5a$ , the authors make the further transformation  $y = z(-m/a)^{1/5}$  to obtain

$$(2) \quad z^5 = pz^3 + qz^2 + rz + 1.$$

The authors tabulate, on pp. 181–201, just a single real root, namely, the smaller or larger positive root of (2) according as  $p + q + r$  is negative or positive, for each of  $p$ ,  $q$ ,  $r = -10(1)10$ , to 5D (6S for over 60 percent of the entries). When  $p + q + r = 0$ ,  $z = 1$  is a positive root of (2).

For interpolation in (1) for nonintegral  $p$  and  $q$ , the authors give a 4-point bilinear formula in the fractional portions of  $p$  and  $q$ , with two examples. For interpolation in (2) for nonintegral  $p$ ,  $q$  and  $r$ , the authors give an 8-point trilinear formula in the fractional portions of  $p$ ,  $q$  and  $r$ , with one example. However, no mention is made of the accuracy attained by those interpolation formulas.

The introductory text consists of eight small pages for (1), two pages for (2), and a prefatory note by A. Zavrotsky.

The computation was performed on an IBM 1620 system in the Electronics Center of Computation of the University of the Andes, using FORTRAN programs for calculation and printout. For the cubic, Cardan's formulas were employed; for the quintic, Horner's method. Altogether, 115 hours were required for the computation.

The text contains a brief historical note mentioning 16 other tables for solving cubics, just by author, place, and year.

In view of the statement in the preface that it is believed that there is not a single error in the thousands of digits comprising the table, the defective page 142, where there is no printout of the imaginary part of the complex root of (1) for  $p = 55, 56, 57, 58$  and  $q = 0(1)50$ , may be only in the reviewer's copy.

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40[2.30, 2.45, 8, 9, 12].—D. E. KNUTH, *The Art of Computer Programming—Errata et Addenda*, Report STAN-CS-71-194, Stanford University, January 1971.

Since Volumes I and II of Knuth's *The Art of Computer Programming* have been so well received, (see our reviews RMT 81, v. 23, 1969, pp. 447–450, and RMT 26, v. 24, 1970, pp. 479–482), it seems desirable to call attention to the extensive changes offered here. A July 1969 "second printing" of Vol. I already included "about 1000 minor improvements". A list of these changes is available from the author.

The present 28 pages of small print include  $7\frac{1}{2}$  more pages of changes in Vol. I,